2020 年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE
GOVERNMENT (MEXT) SCHOLARSHIP 2020

学科試験問題
EXAMINATION QUESTIONS

高等専門学校留学生
COLLEGE OF TECHNOLOGY STUDENTS

数学

MATHEMATICS

注意　☆試験時間は 60 分

PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES

	Nationality		No.			
MATHEMATICS	Name	(Please print full name, underlining family name)			Marks	

1   Answer the following questions and write your answers in the boxes provided.

1) Solve the equation $x^3 - 3x - 2 = 0$.

$$x =$$

2) Solve the equation $\cos 2x + 4\cos x + 3 = 0 \ (0 \le x \le 2\pi)$.

$$x =$$

3) Express $\left| \pi - \dfrac{20}{7} \right| + \left| \pi - \dfrac{23}{7} \right|$ without the absolute value symbols.

4) Solve the equation $3^{2x+2} - 10 \cdot 3^x + 1 = 0$.

$$x =$$

5) Solve the inequality $\log_2 x + \log_2(x - 1) > \log_2(x + 3)$.

6) Calculate the area of the triangle whose vertices are $(1, 1, 1)$, $(-1, 0, 1)$ and $(1, -1, -1)$ in the xyz-space.

7) Let $\alpha : x - 3y + 2z + 7 = 0$. Let l be the line which is perpendicular to α and goes through the point $(0, 0, 0)$. Find the intersection point (x, y, z) of α and l.

$$x = \qquad y = \qquad z =$$

8) Find all tangent lines to the curve $y = x \sin x$ which goes through the point $(0, 0)$.

9) Calculate the infinite series $\displaystyle\sum_{n=1}^{\infty} a_n$ for the sequence $\{a_n\}$ defined as follows:

$$a_1 = 2, \ 2a_{n+1} + a_n = 0.$$

10) Calculate $\displaystyle\lim_{x \to -0} \frac{1 - \cos 4x}{x \, |x|}$.

11) Let $f(x) = \log_{\sqrt{e}} \sqrt{x+1}$. Calculate $f'(x)$.

$$f'(x) =$$

12) Calculate $\displaystyle\int_{-\pi}^{\pi} e^{2x} \sin x \, dx$.

2   For real numbers a, b, $(b > 0)$, let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $A = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ satisfy $A^2 + 2A = -4I$. Answer the following questions and write your answers in the boxes provided.

1) Find a and b.

$$a = \qquad\qquad b =$$

2) Calculate A^{-1}.

$$A^{-1} = \left(\qquad\qquad\right)$$

3) Calculate $(A^{-1})^{3n}$.

$$(A^{-1})^{3n} = \left(\qquad\qquad\right)$$

3 Let $C_1 : y = x^2 + 2x$ and $C_2 : y = x^2 - 4x + 6$. Answer the following questions and write your answers in the boxes provided.

1) Find the intersection point (x, y) of C_1 and C_2.

$$x = \qquad\qquad y =$$

2) Find the common tangent line l of C_1 and C_2.

3) Calculate the area of the region bounded by C_1, C_2 and l.